Foundations of Discrete Mathematics
COT 2104
Summer 2008

Homework 1

NOTE: This homework will be collected in three stages. The corresponding deadline appears before each set of questions. The final grade of this homework will covered the results of the three stages.

Deadline: 05/12/08 (at the beginning of class)

1. Classify each of the following statements as true or false and explain your answers.
   a) "$4 \neq 2 + 2$ and $7 < \sqrt{50}$."
   b) "$4 = 2 + 2 \rightarrow 7 < \sqrt{50}$."
   c) "$4 = 2 + 2 \leftrightarrow 7 < \sqrt{50}$."
   d) "$4 \neq 2 + 2 \leftrightarrow 7 < \sqrt{50}$."
   e) "The area of a circle of radius r is $2\pi r$ or its circumference is πr^2."
   f) "$2 + 3 = 5 \rightarrow 5 + 6 = 10$."

2. Write down the negation of each of the following statements in clear and concise English. Do not use the expression It is not the case that" in your answers.
   a) x is a real number and $x^2 + 1 = 0$.
   b) Every integer is divisible by a prime.
   c) There exist a, b, and c such that $(ab)c \neq a(bc)$.
   d) For every $x > 0$, $x^2 + y^2 > 0$ for all y.
   e) There exists an infinite set whose proper subsets are all finite.

3. Write down the converse and the contrapositive of each of the following implications.
   a) $x^2 = 1 \rightarrow x = \pm 1$.
   b) $ab = 0 \rightarrow a = 0$ or $b = 0$.
   c) If ΔBAC is a right triangle, then $a^2 = b^2 + c^2$.

4. Rewrite each of the following statements using the quantifiers "for all" and "there exists" as appropriate.
   a) For real x, 2^x is never negative.
   b) There are infinitely many primes.
   c) All positive real numbers have real square roots.

5. What is the hypothesis and what is the conclusion in each of the following implications?
   a) The square of the length of the hypotenuse of a right-angled triangle is the sum of the squares of the lengths of the other two sides.
   b) All primes are even.

6. Determine whether or not the following implication is true.
   "x is an even integer $\leftrightarrow$ x + 2 is an even integer."

7. Let n be integer greater than 1 and consider the statement "$\mathcal{A}$: $2^n - 1$ prime is necessary for n to be prime."
   a) Write $\mathcal{A}$ as an implication.
   b) Write $\mathcal{A}$ in the form "p is sufficient for q."
   c) Write the converse of $\mathcal{A}$ as an implication.
   d) Determine whether the converse of $\mathcal{A}$ is true or false.

8. Let a and b integers. By examining the four cases
   I. a, b both even.
   II. a, b both odd.
   III. a even, b odd.
   IV. a odd, b even

   Find a necessary and sufficient condition for $a^2 - b^2$ to be odd.

9. Let x be a real number. Find a necessary and sufficient condition for $x + 1/x \geq 2$. Prove your answer.

10. Prove that if n is an odd integer, there is an integer m such that n = 8m + 1 or n = 8m + 3or n = 8m +5 or n = 8m + 7 (You may use the results of Exercise 22 on page 16).

Deadline: 05/19/08 (at the beginning of class)

11. Construct a truth table for each of the following compound statements
    a) $(p \wedge q) \vee ((\neg p) \rightarrow q)$
    b) $\neg (p \wedge (q \vee p)) \leftrightarrow p$

12. If $p \rightarrow q$ is false, determine the truth value of $(p \wedge (\neg q)) \vee ((\neg p) \rightarrow q)$
    $[p \rightarrow q)$

13. Determine the truth value for $[p \rightarrow (q \wedge (\neg r))] \vee [r \leftrightarrow ((\neg s) \vee q)]$ where p, q, r, and s are all false.

14. Show that $q \rightarrow (p \rightarrow q)$ is a tautology.

15. Determine whether or not the following argument is valid
    a. $p \rightarrow q$
       $r \rightarrow q$

       $r \rightarrow p$

b. $p \rightarrow q$

$(q \vee (\neg r)) \rightarrow (p \wedge s)$

$s \rightarrow (r \vee q)$

Deadline: 06/02/08(at the beginning of the test)

16. List the (distinct) elements in each of the following sets:
    a) $\{x \in Z \mid xy = 15 \text{ for some } y \in Z\}$
    b) $|x + y| \; x \in \{-1, 0, 1\}, y \in \{0, 1, 2\}$

17. Determine whether each of the following statements is true or false.
    a) $\emptyset \subseteq \{\emptyset\}$
    b) $\emptyset \in \emptyset$
    c) $\emptyset \in \{\emptyset\}$

18. Let $A = \{x \in N \mid x < 7\}$, $B = x \in Z \mid |x - 2| < 4 \}$ and $C = \{ x \in N \mid x^3 - 4x = 0\}$.
    a) Find $A \cup C$, $B \cap C$, $B \setminus C$, $A \oplus B$, $C \times (B \cap C)$, $(A \setminus B) \setminus C$, $A \setminus (B \setminus C)$, and $(B \cup \emptyset) \cap \{\emptyset\}$.

19. For $A = \{a, b, c, \{a, b\}\}$, find
    a) $\{\emptyset\} \setminus \mathcal{P}(A)$
    b) $A \setminus \emptyset$
    c) $\emptyset \setminus A$
    d) $(\{a, b, c\} \cup \{A\}) \setminus A$

20. Explain why the following binary relations on $S = \{1, 2, 3\}$ is not an equivalence relation on S.

    a) $R = \{(1, 1), (2,2), (3, 3), (2, 1),(1, 2), (2, 3), (3, 1), (1, 3)\}$

21. Determine, with reasons, whether or not each of the following defines an equivalence relation on the set A.
    a) A is the set of all circles in the plane: $a \sim b$ if and only if a and b have the same center.
    b) A is the set of all straight lines in the plane: $a \sim b$ if and only if a is parallel to b.